A general bistatic SAR focusing algorithm for azimuth variant
and invariant configurations

Qurat Ul-Ann, Otmar Loffeld, Holger Nies, Robert Wang*

University of Siegen, Siegen, Germany, ul-ann@ipp.zess.uni-siegen.de
*Chinese Academy of Sciences, Beijing, China

Keywords: Bistatic Synthetic Aperture Radar (SAR), bistatic
point target reference spectrum, method of stationary phase,
scaled inverse Fourier transformation

Abstract

This paper presents a general bistatic SAR focusing algorithm
for azimuth variant and invariant configurations. The
approach used in this paper is based on Loffeld’s Bistatic
Formula (LBF). We considered different azimuth
contributions of the transmitter and the receiver phase terms
in the derivation of the Bistatic Point Target Reference
Spectrum (BPTRS). An efficient focusing algorithm is
implemented using Scaled Inverse Fourier Transformation
(SIFT) and is verified with focusing results of azimuth variant
and invariant configurations.

1 Introduction

Bistatic SAR has gained much attention over the past years
[1-18]. A general bistatic SAR configuration offers a complex
geometry, with transmitter and receiver located at different
platforms, moving in different directions and with different
velocities. It not only allows different data acquisition
geometries but also provides more information about the
imaging scene.

Several techniques have been used for the derivation of
BPTRS [1-18]. [2] elucidates a two stage approach, first
preprocessing the raw data and subsequently using a
monostatic processor. A technique called “dip move out
(DMO)” is used in [1] and further considered in [9]. In [17],
an Omega-k algorithm is used for bistatic focusing. In [18],
a range Doppler algorithm is proposed for the analytical
spectrum and is derived using the Method of Series Reversion
(MSR) for azimuth invariant configurations. The bistatic
focusing for tandem and Translationally Invariant (TI)
configurations is considered in [7] and is extended to general
configurations in [8], where a 2-D SIFT is used to focus
bistatic SAR data. In [8], different azimuth contributions of
the transmitter and receiver phase terms have not been taken
into account, therefore it does not work well for spaceborne/airborne configurations. The BPTRS based on
LBF is considered in this paper, which consists of a quasi
monostatic and a bistatic deformation phase term [10]. In
[15], the linearization of BPTRS accommodates both the
bistatic deformation and the quasi monostatic phase terms. It
involves many phase terms and computationally complex
range and azimuth variant modulation and scaling terms, in
spite of the fact that the contribution of linearized bistatic
deformation term is negligible towards them.

This paper is structured as follows: In the next section, we
describe the geometry and signal model of a general bistatic
SAR configuration. The point target reference spectrum based
on different azimuth contributions of the transmitter and the
receiver phase terms is considered. The spectrum of the
complete scene is derived and a focusing algorithm is
provided in section 3. We analyze azimuth invariant and
variant configurations in sections 4 and 5 respectively and
focusing results are provided for both cases. Finally, some
conclusions are drawn in the last section.

2 Geometrical model and the bistatic point
target reference spectrum

A geometrical model of a general bistatic SAR configuration
is shown in figure 1. The azimuth time is denoted by \( \tau \).
\( v_t, v_r \) are the velocities of transmitter and receiver. \( \tau_{0t}, \tau_{0r} \)
are the azimuth times, and \( R_{0t}, R_{0r} \) are the slant ranges, when
transmitter and receiver are at their closest distances from the
point target \( PT(R_{0t}, \tau_{0t}) \). In our approach [10], all the terms
are defined with respect to the receiver’s coordinates
\( (R_{0r}, \tau_{0r}) \).

![Figure 1: General Bistatic SAR Geometry](image)

The complete bistatic slant range history is the sum of
transmitter and receiver slant range histories and is given as:

\[
R_b(\tau, R_{0t}, \tau_{0t}) = R_b(\tau, R_{0r}, \tau_{0r}) + R_b(\tau, R_{0t}, \tau_{0r})
\]

\[
R_b(\tau) = \sqrt{R_{0t}^2 + v_t^2 (\tau - \tau_{0t})^2}; \quad R_b(\tau) = \sqrt{R_{0r}^2 + v_r^2 (\tau - \tau_{0r})^2}
\]  (1)
The received signal is a time delayed replica of the transmitted signal. The point target response in the low pass domain is written as:

\[ g_t(t, \tau, R_{to}, \tau_{to}) = \sigma(R_{to}, \tau_{to}) w(\tau - \tau_{to}) s_t(t - \tau_{to}) e^{-j2\pi f_0 \tau_{to}} \]

\[ t_0(\tau) = \left[ R_t(\tau) + R_r(\tau) \right] / c_0 \]

(2)

\[ \sigma(R_{to}, \tau_{to}) \] is the backscattering coefficient, \( t \) denotes the range time and \( w(\tau - \tau_{to}) \) is the azimuth time window with center time \( \tau_{to} \). \( s_t(t) \) represents the transmitted signal. Now, after performing the Fourier transformation over the range and azimuth time, we get:

\[ G_i(f, f_r, R_{to}, \tau_{to}) = \sigma(R_{to}, \tau_{to}) S_i(f) \]

\[ \int_{-\infty}^{\infty} w(\tau - \tau_{to}) e^{-j2\pi(f+\nu)(\tau - \tau_{to})/c_0} d\tau \]

(3)

\[ \phi_b \] is the bistatic phase term, which is the sum of transmitter \( \phi_t \) and receiver \( \phi_r \) phases. Here, we consider different azimuth contributions of transmitter and receiver in the derivation of the BPTRS, given in [13, 14]. The final expression of the BPTRS is obtained as:

\[ G_i(f, f_r, R_{to}, \tau_{to}) = \frac{\sigma(R_{to}, \tau_{to}) w(\tau - \tau_{to}) S_i(f)}{A_{mp}(f, f_r)} e^{-j2\pi (f + \nu)(\tau - \tau_{to})/c_0} \]

(4)

Where, \( \psi_{ot}, \psi_{ort}, A_{mp} \) are the quasi-monostatic, bistatic deformation phases and amplitude terms respectively [14].

3 A general focusing algorithm

The spectrum of a complete scene is the sum of the reflected signals from all point targets and is expressed as [8]:

\[ W(f, f_r) = \int_{-\infty}^{\infty} G_i(f, f_r, R_{to}, \tau_{to}) R_{to} dr_0 d\tau_{to} \]

(5)

The next step is to perform range compression, amplitude correction and bistatic deformation phase term compensation and is represented as:

\[ W'(f, f_r) = W(f, f_r) S'_i(f) e^{j\psi_{ort}} A_{mp}(f, f_r) \]

(6)

The complete scene spectrum after performing the above step is written as:

\[ W'(f, f_r) = \int_{-\infty}^{\infty} G_i(f, f_r, R_{to}, \tau_{to}) R_{to} dr_0 d\tau_{to} \]

(7)

Here, \( \psi_{cs} \) is the remaining phase term of the complete scene and is expressed as:

\[ \psi_{cs}(f, f_r, R_{to}, \tau_{to}) = \psi_{cs}(f) + j2\pi R_{to} F_p^2 [f + f_0] + \psi_{ort} \]

(8)

The frequency histories of transmitter and receiver \( F_t, F_r \) and the weighting factor \( \mu \) are given in [13, 14]. In spaceborne/airborne configurations, \( \mu \) is used for different azimuth contributions of transmitter and receiver to the bistatic phase. From (4), it is evident that BPTRS depends non-linearly on the four space position variables i.e. \( \tau_{ot}, \tau_{ort}, R_{to}, R_{ort} \). We linearize the spectrum by expressing \( \tau_{ot}, R_{ort} \) in terms of the receiver’s coordinates \( \tau_{or}, R_{or} \).

\[ \begin{bmatrix} \tau_{ot} \\ \tau_{ort} \end{bmatrix} = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \end{bmatrix} \begin{bmatrix} \tau_{or} \\ R_{or} \end{bmatrix} \]

(9)

Here, \( e_{11}, e_{12}, e_{13} \) are the linearization parameters [8, 15]. The linear expression of complete scene phase term is represented as:

\[ \psi_{ot}(f, f_r) = \sum_{n=0}^{2} \psi_{ort}^{(n)}(f) R_{ort}^{(n)} \]

Now a range variable \( r = R_{or} - R_{ot} \) is introduced, which is the difference between \( R_{or} \) and \( R_{ot} \). Here, \( R_{or} \) is the closest range from the scene centre to the receiver’s track. The first range phase term is space invariant and is given as:

\[ \psi_{ort}^{(1)}(f) = \sum_{n=0}^{1} \psi_{ort}^{(n)}(f) R_{ort}^{(n)} \]

(10)

The second and third phase terms of equation (10) are azimuth and range variant and are expressed as:

\[ \psi_{ort}^{(2)}(f) = \psi_{ort}^{(2)}(f) R_{ort}^{(2)} \]

(11)

\[ \psi_{ort}^{(3)}(f) = \psi_{ort}^{(3)}(f) R_{ort}^{(3)} \]

(12)

The range and azimuth variant phase terms of the above equation are expanded using a first order Taylor expansion at range \( f \) and azimuth \( f_r \) frequencies respectively, we get the following expressions:

\[ \psi_{ort}^{(1)}(f, f_r) = A_{ot}(f, f_r) + B_{ot}(f, f_r) f \]

(13)

\[ \psi_{ort}^{(2)}(f, f_r) = A_{ot}(f) + B_{ot}(f, f_r) \]

(14)

Where, \( A_{ot}, A_{ot} \) are shifts and \( B_{ot}, B_{ot} \) are scaling factors in range and azimuth respectively. The scaling and shift factors are given as:

\[ A_{ot}(f) = \frac{2\pi R_{ort} R_{ot} R_{ort} R_{ot} R_{ort}}{c_0} \left[ 1 - \frac{(2 - \mu)^2}{4 R_{ort}^2 R_{ot}^2} \right] \]

(15)

We can see from the above equations that scaling and shift factors in range depend on azimuth frequency. The scaling factor in azimuth is constant, while the shifting factor depends on range frequency. The spectrum given in the equation (7) is simplified as:

\[ W'(f, f_r) = e^{j\psi_{cs}(f, f_r)} \int_{-\infty}^{\infty} \sum_{n=0}^{2} \psi_{ort}^{(n)}(f) R_{ort}^{(n)} \]

(16)
The final spectrum is represented in terms of the back-scattering coefficients spectrum, which is scaled and shifted in range and azimuth as:

$$ W(f, f_0) = W^*(f, f_0) e^{j\psi_{CS0}(f, f_0)} e^{-j\left(\theta_{2R}(f) + \theta_{2A}(f)\right)}$$

Now the next step is to compensate the shift and scaling in range and azimuth. A SIFT will be used to convert the scaled spectrum into its non-scaled counterpart [11]. Firstly, we perform a SIFT in range and scaling is corrected as:

$$ \sigma(r, A_R(f) + B_R(f)) = H_{2R_0}(r, f) \int W^*(f, f_0) H_{1R_0}(f, f_0) df$$

Where,

$$ H_{1R_0}(f, f_0) = e^{j2\pi \theta_{2R}(f_0)/f}; H_{2R_0}(r, f) = B_R(f) e^{j2\pi \theta_{2R}(f)}$$

After that a 1D-FFT in range is required. Secondly, we perform the SIFT in azimuth and the scaling is corrected as:

$$ \sigma(f, r_{0R}) = H_{1A_0}(f, f_0) \int \sigma(f, A_0(f) + B_0(f)) df$$

Where,

$$ H_{1A_0}(f, f_0) = e^{j2\pi \theta_{2A}(f_0)/f}; H_{2A_0}(r, f) = B_A(f) e^{j2\pi \theta_{2A}(f)}$$

The above equation represents the backscattering coefficient in range frequency and azimuth time domain. Performing a 1D-IFFT in range gives the complex image $$ \sigma(r, \tau_{0A}) $$ . The shift and scaling factors used in the equations (18) and (20) depend on the weighting factor $$ \mu $$ . The processing steps of the bistatic SAR focusing algorithm are shown in figure 2.

4 Azimuth invariant configurations

The bistatic SAR configurations can be categorized into two groups; azimuth variant and azimuth invariant. In this section, we consider Azimuth Invariant Configurations (AIC). The baseline between the transmitter and the receiver remains constant during the flight in AIC. Examples of such configurations are tandem configurations (the transmitter and the receiver flying on the same path with constant baseline) and the TI configurations (the transmitter and the receiver flying parallel to each other with constant baseline) [6, 12, 16].

The AIC are also called constant offset configurations. The system parameters $$ a_0 = (\tau_{0T} - \tau_{0R}) $$ and $$ a_2 = (R_{0T} / R_{0R}) $$ which represent the azimuth time difference and slant range ratio of the transmitter and the receiver, at the point of closest approach, are constant. As $$ a_0, a_2 $$ are constant, the processing algorithm given in section 3 can be simplified accordingly. Azimuth scaling correction is also not necessary for constant offset configurations. As transmitter and receiver contribute equally towards the bistatic phase term, we use a weighting factor of $$ \mu = 1 $$ [14].

4.1 Bistatic Airborne Experiment

We consider a bistatic airborne experiment performed by Fraunhofer-Institute of High Frequency Physics FHR (former FGAN) in November 2003 [3, 5, 6]. The sensor AER-II is used as transmitter and PAMIR is used as receiver. Both platforms had parallel trajectories and had bistatic angle of 13°. The imaging site was Oberndorf am Lech, Germany. We use our proposed processing algorithm for focusing of the raw data obtained during this experiment. The image obtained is shown in figure 3 and some important parameters are given in table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Transmitter</th>
<th>Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>98 m/s</td>
<td></td>
</tr>
<tr>
<td>Pulse Duration</td>
<td>3 $\mu$s</td>
<td></td>
</tr>
<tr>
<td>Common Bandwidth</td>
<td>300 MHz</td>
<td>10.17 GHz</td>
</tr>
<tr>
<td>Carrier Frequency</td>
<td>10.13 GHz</td>
<td>10.17 GHz</td>
</tr>
<tr>
<td>PRF</td>
<td>1250 Hz</td>
<td></td>
</tr>
<tr>
<td>Range Sampling Rate</td>
<td>400 Msamples/s</td>
<td></td>
</tr>
<tr>
<td>Slant Ranges</td>
<td>2906 m</td>
<td>3590 m</td>
</tr>
</tbody>
</table>

Table 1: Azimuth Invariant Configuration.

![Figure 2: Block Diagram of the Processing Algorithm](image)

![Figure 3: Focused Image - Azimuth Invariant Configuration](image)
Azimuth variant configurations

Azimuth Variant Configurations (AVC), also called general configurations, are those where the baseline between the transmitter and the receiver varies over azimuth time. Hybrid configurations are an example of AVC. In hybrid configurations, a satellite is used as transmitter and an aircraft is used as receiver. Because of the huge difference between the velocities and altitudes of transmitter and receiver, we consider the weighted azimuth contribution of transmitter and receiver phases in LBF [14].

5.1 Hybrid experiment 1

A hybrid experiment was performed by Frauenhofer-FHR in July 2008, using PAMIR as receiver and TerraSAR-X satellite as transmitter [4]. The test site chosen was Pommersfelden, Germany. Both transmitter and receiver have X-band phased array antennas. Transmitter was operated in sliding spotlight mode and receiver in stripmap mode. The receiver’s azimuth beam width was increased to increase the azimuth scene extent, which on other hand decreases the SNR. The PRF of transmitter is approximately three times larger than that of receiver [4]. The fractional mismatching of PRF introduces constant drift in the range lines along azimuth of the raw data and it needs to be corrected before applying frequency domain processing algorithm. Some important parameters of this experiment are given in table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Transmitter</th>
<th>Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>7600 m/s</td>
<td>95 m/s</td>
</tr>
<tr>
<td>Altitude</td>
<td>515 km</td>
<td>3300 m</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>150 MHz</td>
<td></td>
</tr>
<tr>
<td>Azimuth Beam width</td>
<td>0.33°</td>
<td>6°</td>
</tr>
<tr>
<td>Incidence Angle</td>
<td>47°</td>
<td>70°</td>
</tr>
<tr>
<td>PRF</td>
<td>3798.61 Hz</td>
<td>1266.22 Hz</td>
</tr>
<tr>
<td>Wavelength</td>
<td>3.1 cm</td>
<td></td>
</tr>
<tr>
<td>Slant Ranges</td>
<td>723 km</td>
<td>9900 m</td>
</tr>
</tbody>
</table>

Table 2: Azimuth Variant Configuration.

The raw data was down sampled upon receiving and is therefore up sampled before processing to avoid aliasing in azimuth. The raw data is then processed using our proposed focusing algorithm. The image obtained is shown in figure 4. We processed the same data set using a back propagation algorithm to compare the processed results with the back propagation algorithm are shown in figure 5. Comparing the results we can say that our proposed frequency domain algorithm provides good quality images in considerably shorter time and is approximately 30 times faster than the back propagation algorithm.

Figure 4: Focused Image Using Frequency Domain Algorithm

Figure 5: Focused Image Using Back Propagation Algorithm
5.2 Hybrid experiment 2

We consider another hybrid experiment, performed by Frauenhofer-FHR in 2009, using PAMIR as receiver and TerraSAR-X satellite as transmitter. The imaging site was Niederweidbach, Germany. In this experiment, a reduced receiver azimuth beam width was used to achieve a better SNR. The azimuth scene extent was increased by using a double sliding spotlight mode [4]. PAMIR and TerraSAR-X operated in inverse sliding and sliding spotlight mode respectively. Both transmitter and receiver have X-band phased array antennas with the ability of beam steering. The important parameters of experiment are given in table 3.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Transmitter</th>
<th>Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>7600 m/s</td>
<td>100 m/s</td>
</tr>
<tr>
<td>Altitude</td>
<td>515 km</td>
<td>3400 m</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>300 MHz</td>
<td></td>
</tr>
<tr>
<td>Azimuth Beam width</td>
<td>0.33°</td>
<td>2.5°</td>
</tr>
<tr>
<td>Incidence Angle</td>
<td>55.3°</td>
<td>65°</td>
</tr>
<tr>
<td>PRF</td>
<td>4374.45 Hz</td>
<td>1458.15 Hz</td>
</tr>
<tr>
<td>Range Sampling Rate</td>
<td>400 Msamples/s</td>
<td></td>
</tr>
<tr>
<td>Slant Ranges</td>
<td>844.1 km</td>
<td>8000 m</td>
</tr>
</tbody>
</table>

Table 3: Azimuth Variant Configuration (Niederweidbach)

The raw data of this experiment is processed using our proposed focusing algorithm. The image obtained is shown in figure 6.

Figure 6: A Focused Image by Proposed Algorithm

6 Conclusions

This paper presented a general SIFT processing algorithm for azimuth variant and invariant configurations assuming different azimuth phase contributions of transmitter and receiver in LBF. TI and hybrid configurations have been investigated as examples of azimuth invariant and variant configurations, respectively. Focusing results of real data from airborne/airborne and hybrid experiments show good quality images. A comparison of the results with back propagation algorithm, further verifies our frequency domain processing algorithm.

Acknowledgements

The authors are thankful for the highly effective collaboration between ZESS (Siegen University) and Frauenhofer FHR.

References


