Development of a Deeply-Coupled GPS/INS Integration Algorithm using Quaternions

Yuhong Yang, Junchuan Zhou, Holger Nies, Otmar Loffeld
Center for Sensor Systems (ZESS)
University of Siegen
Siegen, Germany

Stefan Knedlik
iMAR GmbH
Im Reihersbruch 3
St. Ingbert, Germany

Abstract—A deep-coupled Global Positioning System (GPS) / Inertial Navigation System (INS) integration algorithm is proposed in this paper. The mathematical system process and observation models are provided. Due to the nonlinearity of the system models, an Extended Kalman Filter (EKF) is employed, which uses quaternions as the representation of attitude. The integrated algorithm is tested using the IFEN GPS radio frequency (RF) signal simulator. Both static and dynamic scenarios are simulated. Numerical results are compared and analyzed.

Keywords—Deeply-coupled; GPS/INS; Quaternions

I. INTRODUCTION

Deeply-coupled GPS/INS integration research has drawn more attention in the recent few years. Traditional GPS/INS integration employs a loosely or a tightly-coupled architecture. One example of different implementations of GPS/INS integration systems is illustrated in Fig.1.

![GPS/INS integration architectures.](image)

Fig. 1. GPS/INS integration architectures.

The loosely-coupled integration has a decentralized estimation architecture, which uses the output information of the navigation solutions from a GPS receiver and an INS. The main advantages of using loosely-coupled integration are: a) the system observation model is simpler, and accordingly it requires much less computational burden; b) the number of measurement inputs (i.e., position and velocity) for the Kalman filter is fixed; c) redundant GPS navigation solutions are available. However, the disadvantages are: a) in case of using two separate KFs (i.e., one for GPS navigation processing, and the other for integration purpose), it opens the possibility of presenting instable navigation solutions caused by mutual feedbacks of estimation errors, which is coined as cascaded filtering problem; b) usually more than 4 satellites are required to maintain a GPS navigation solution.

Unlike the loosely-coupled manner, in a tightly-coupled integration, only a centralized KF is used. The pseudorange and delta range (or Doppler) measurements are directly used in the filter. The advantages of applying the tightly-coupled integration are mainly in the following aspects: a) the cascaded filtering problem arising through the mutual feedbacks of the estimation errors between two separate KFs is eliminated; b) the system does not require a GPS navigation solution to aid the INS. Therefore, even with less than 4 satellites in view, the remaining satellite based measurements can still be used in the algorithm, which promotes the robustness of the navigation system; c) all systematic errors and noise sources of the distributed sensors are modeled in the same filter, which ensures that all error correlations are accounted for. Nevertheless, the disadvantage of tightly-coupled approach arises in the increased dimension of the observation vector with respect to the loosely-coupled manner. And the number of GPS measurements as inputs to the Kalman filter is varying in practical applications, which depends on the signal environments.

It is known that in a tightly-coupled system, all GPS tracking channels are working independently. In this work, we move one step forward to implement the deeply-coupled integration. The algorithm we proposed uses quaternions as the representation of attitude. This work is based on a software-defined GPS receiver we developed. The former work on this topic can be found in [1-3]. In this system, an EKF is employed to fuse the outputs (i.e., code phase error and frequency error) from the receiver tracking loops and Inertial Measurement Unit (IMU) observables to enhance GPS tracking performance, which gives robust and smoothed navigation results.

II. IMPLEMENTATION OF A DEEPLY-COUPLED INTEGRATION SYSTEM

A. System Architecture

The realization of a deeply-coupled integration can be mainly divided into two categories [4]. One way directly uses the GPS receiver’s baseband correlator outputs (i.e., Is and Qs) as the measurements to fuse with the output of IMU. The other way is based on the design of a vector tracking loop, which was first proposed by Spilker [5]. He used nonlinear discriminator outputs as measurements. In this paper, our development is based on the second approach. The high level architecture of the algorithm is illustrated in Fig. 2. We employ an IMU, which contains 3-axis gyroscope and 3-axis accelerometer.
Discriminator is given in (2). For a detailed derivation, 
\(w_{k,j,k} \) and \(\rho_{z}\) and \(\rho_{k}\). It is used to represent the rotation from the 
\(\phi\) drift error [m/s]; \(f\) frequency errors [m/s] at epoch \(k\) for certain tracking loops; 
whether the channels are no longer working independently, but 
are coupled to a common navigation solution. Thus, with the 
assistance from the IMU, the channels can principally help 
each other in the case of GPS signal attenuation conditions.

In Fig. 2, the acquisition block (at the left side of the plot) is 
used to acquire the incoming signals from the available 
satellites, and to provide code phase and coarse Doppler 
frequency to the tracking loop. In the tracking loop, the carrier 
frequency is refined to several Hz precision. Unlike the 
traditional GPS receiver, our deeply-coupled system uses a 
global integration Kalman filter for all channels. For this 
reason, the channels are no longer working independently, but 
are coupled to a common navigation solution. Thus, with the 
assistance from the IMU, the channels can principally help 
each other in the case of GPS signal attenuation conditions.

In such a deeply-coupled system, the essential 
relationships are the connection between the code phase error 
and the position, clock error, and the connection between the 
carrier frequency error and velocity, clock drift. They have the 
mathematical relations as shown in (1). Details can be also 
found in [6]. Details can be also found in [6].

\[
z_{j,k}^\phi = LOS_{j,k}^T \cdot \delta \hat{p}_{k} + \delta t_{b,k} + w_{j,k}^\phi \\
z_{j,k}^f = LOS_{j,k}^T \cdot \delta \hat{v}_{k} + \delta t_{d,k} + w_{j,k}^f 
\tag{1}
\]

where \(z_{j,k}^\phi\) and \(z_{j,k}^f\) are the code phase error [m] and carrier 
frequency errors [m/s] at epoch \(k\) for certain tracking loops; 
\(\delta \hat{p}_{k}\) and \(\delta \hat{v}_{k}\) are the estimated position error and velocity 
ear error at epoch \(k\); \(LOS_{j,k}\) represents the unit line of sight 
vector from the receiver to the \(j\)-th satellite; \(\delta t_{b,k}\) is the 
receiver clock bias in distance, and \(\delta t_{d,k}\) denotes the clock 
drift error [m/s]; \(w_{j,k}^\phi\) and \(w_{j,k}^f\) represent the white Gaussian 
noise errors.

B. INS Principle

Inertial navigation is based on Newtonian physics and is 
affected by gravity. That is, the object will remain in uniform 
motion unless disturbed by an external force. It involves a 
bias of inertial measurements, mathematics, control system 
design and geodesy [7]. The external force generates 
acceleration on the object, which can be measured by the 
inertial sensor. After the integration of the measured 
advertisements, under consideration of measurements from 
gyrosopes, the change in velocity and position with respect to 
the initial conditions can be determined. A conventional IMU 
consists of three gyroscopes for measuring angular rates and 
three accelerometers for measuring accelerations. They are 
mounted in triads so that the sensitive axes of sensors are 
mutually orthogonal, setting up a Cartesian reference frame. 
For a strapdown inertial sensor platform, the inertial sensor is 
rigidly mounted to the structure of the vehicle. The sensor raw 
data are processed to yield navigation solutions (i.e., position, 
velocity, and attitude over time), and this process is named 
strapdown processing. The INS strapdown processing models 
in continuous and discrete time domains are given in the next 
section using quaternions as the representation of attitude.

C. INS Strapdown and System Process Models

We will not use Euler angles as representation of attitude 
due to the inherent problem of singularity [7]. In this paper, 
the quaternion vector is denoted as \(\vec{q} = [q_x, q_y, q_z]^T\) , where 
\(\vec{q} = [q_1, q_2, q_3, q_4]^T\). It is used to represent the rotation from the 
navigation frame to the body frame. The attitude differential equation in terms of the 
quaternion vector \(\vec{q}\) is given in (2). For a detailed derivation, 
the reader is referred to [8, 9].

\[
\vec{q} = \frac{1}{2} Q_{\omega_{ib}} \vec{q} \quad \text{with} \quad Q_{\omega_{ib}} = \begin{bmatrix}
0 & -\omega_{ib}^b & \omega_{ib}^b \\
\omega_{ib}^b & 0 & -\omega_{ib}^b \\
-\omega_{ib}^b & \omega_{ib}^b & 0
\end{bmatrix}
\tag{2}
\]

where \(\omega_{ib}^b\) is the angular rate measurement vector from the 
navigation frame to the body frame, expressed in the body 
frame, which is equal to:

\[
\omega_{ib}^b = \omega_{ib}^b - \omega_{ib}^b 
\tag{3}
\]

where \(\omega_{ib}^b\) is the rotational rate vector of the body frame 
relative to the inertial frame, expressed in the body frame (i.e., 
IMU gyroscope raw measurements); \(\omega_{ib}^b\) represents the sum 
of the rotation of the Earth with respect to the inertial frame 
plus the turn rate of the navigation frame with respect to the 
Earth, expressed in the body frame, i.e., \(\omega_{ib}^b = R_b^e \left(\omega_{ib}^b + \omega_{ib}^b\right)\).

Using a low-cost MEMS-based IMU, the Earth rotation is 
usually buried in sensor errors, and cannot be detected. The 
Coriolis and centrifugal terms are not considered in the 
following sections. Moreover, for short distance applications, 
the transport rate is negligible. Considering these effects, we 
have \(\omega_{ib}^b = \vec{0}\). Thus, (3) turns to be:

\[
\omega_{ib}^b = \omega_{ib}^b \Rightarrow \omega_{ib}^b = -\omega_{ib}^b 
\tag{4}
\]

where \(\omega_{ib}^b = [\omega_{ib,x}^b, \omega_{ib,y}^b, \omega_{ib,z}^b]^T\) is the gyroscope raw data, 
resolved in the body frame.
where the navigation frame is expressed using quaternions:

\[ \mathbf{q} = \frac{1}{2} \mathbf{Q}_{\omega_w^b} \mathbf{q} . \]

(5)

with \( \mathbf{Q}_{\omega_w^b} = \begin{bmatrix} 0 & -\mathbf{\omega}_w^b \times & \mathbf{0}^b \times & \mathbf{0}^b \times \end{bmatrix} \).

(6)

The simplified strapdown mechanization model for the IMU can be expressed in the navigation frame, as shown in (6):

\[ \mathbf{p}_w = \mathbf{v}_w, \]

\[ \dot{\mathbf{v}}_w = \mathbf{R}_n^b(\mathbf{q}) \mathbf{f}_b + \mathbf{g}_n \]

\[ \dot{\mathbf{q}} = \frac{1}{2} \mathbf{Q}_{\omega_w^b} \mathbf{q} \]

where \( \mathbf{p}_w \) stands for position and \( \mathbf{v}_w \) is velocity in navigation frame; \( \mathbf{g}_n \) represents gravity indicated in the navigation frame, which is assumed to be approximately constant in the region of interest.

The rotational transformation matrix \( \mathbf{R}_n^b(\mathbf{q}) \) from the body frame to the navigation frame is expressed using quaternions:

\[ \mathbf{R}_n^b(\mathbf{q}) = \begin{bmatrix} q_1^2 + q_2^2 - q_3^2 - q_4^2 & 2(q_2 q_3 - q_1 q_4) & 2(q_1 q_2 + q_3 q_4) & \mathbf{0}^b \\
2(q_2 q_3 + q_1 q_4) & q_1^2 - q_2^2 + q_3^2 - q_4^2 & 2(q_1 q_3 - q_2 q_4) & \mathbf{0}^b \\
2(q_2 q_4 - q_3 q_1) & 2(q_1 q_4 + q_2 q_3) & q_1^2 - q_2^2 - q_3^2 + q_4^2 & \mathbf{0}^b \\
\mathbf{0}^b & \mathbf{0}^b & \mathbf{0}^b & \mathbf{0}^b \end{bmatrix} \]

(7)

The IMU specific force and angular rate measurement errors (e.g., sensor biases) are modeled as constants superseded by random walk, which are given in (8):

\[ \mathbf{f}_b^{bias} = \mathbf{w}_f, \]

\[ \mathbf{\omega}_b^{bias} = \mathbf{w}_w \]

(8)

where \( \mathbf{w}_f \) and \( \mathbf{w}_w \) are assumed to be zero mean, Gaussian distributed.

It is worth mentioning that using (6) as the GPS/INS system propagation model, the IMU incoming measurements should be compensated by the current estimate of the sensor biases before further processing, i.e., \( \mathbf{f}_b = \mathbf{f}_b^0 - \mathbf{f}_b^{bias} - \mathbf{n}_f, \)

\[ \mathbf{\omega}_b = \mathbf{\omega}_b^0 - \mathbf{\omega}_b^{bias} - \mathbf{n}_w \] (\( \mathbf{n}_f \) and \( \mathbf{n}_w \) are the remaining noises which are assumed to be zero-mean, Gaussian distributed).

In the discrete time domain with a sufficiently small time interval (e.g., 0.01s IMU update time), and for low dynamic applications, we have:

\[ \mathbf{p}_{n,k+1} = \mathbf{p}_{n,k} + \mathbf{v}_{n,k} \cdot T \]

\[ \mathbf{v}_{n,k+1} = \mathbf{v}_{n,k} + \left[ \mathbf{R}_n^b(\mathbf{q}_k) \mathbf{f}_b^k + \mathbf{g}_n \right] T \]

\[ \mathbf{q}_{k+1} = \mathbf{q}_k + \left( \frac{1}{2} \mathbf{Q}_{\omega_w^b} \cdot T \right) \mathbf{q}_k \]

\[ \mathbf{f}_{b,k+1}^{bias} = \mathbf{f}_{b,k}^{bias} + \mathbf{w}_{f,k} \]

\[ \mathbf{\omega}_{b,k+1}^{bias} = \mathbf{\omega}_{b,k}^{bias} + \mathbf{w}_{w,k} \]

where “\( T \)” is system propagation time interval.

In (9), a part of the model is nonlinear, e.g., \( \mathbf{R}_n^b(\mathbf{q}) \) contains quadratic terms of quaternion elements. Therefore, the linearization process should be conducted in order to apply Kalman filtering. An EKF is used in this paper, which is based on a first order linearization process on the stochastic system model with the assumption of Gaussian distributed noises. Besides, in the scope of a deeply-coupled integration approach, the receiver clock errors need to be modeled. The range-rate equivalent of the clock drift error is modeled as a constant plus a random walk process, while the range equivalent of the receiver clock bias error is the integral of the clock drift error. Thus, the linearized system propagation model used for GPS/INS deeply-coupled integration is formulated in (10) at the bottom of the page, where error states are employed.
\[
\mathbf{F}_{23,k} = 2 \cdot \begin{bmatrix}
q_1 \hat{f}_x + q_2 \hat{f}_x - q_3 \hat{f}_x & q_2 \hat{f}_x + q_4 \hat{f}_x + q_3 \hat{f}_x \\
q_2 \hat{f}_y - q_3 \hat{f}_y + q_4 \hat{f}_y & q_3 \hat{f}_y - q_4 \hat{f}_y + q_1 \hat{f}_y \\
q_3 \hat{f}_z - q_4 \hat{f}_z + q_2 \hat{f}_z & -q_1 \hat{f}_z + q_4 \hat{f}_z + q_3 \hat{f}_z
\end{bmatrix}
- \begin{bmatrix}
-q_1 \hat{f}_x + q_2 \hat{f}_x - q_3 \hat{f}_x & q_1 \hat{f}_x - q_3 \hat{f}_x + q_2 \hat{f}_x \\
q_2 \hat{f}_y + q_3 \hat{f}_y + q_4 \hat{f}_y & q_3 \hat{f}_y + q_1 \hat{f}_y - q_2 \hat{f}_y \\
q_3 \hat{f}_z + q_4 \hat{f}_z + q_2 \hat{f}_z & q_4 \hat{f}_z - q_2 \hat{f}_z + q_3 \hat{f}_z
\end{bmatrix}
\]
\]

where \( \text{dot} = I_{P1} \cdot I_{P2} + Q_{P1} \cdot Q_{P2} \), \( \text{cross} = I_{P1} \cdot Q_{P2} - I_{P2} \cdot Q_{P1} \).

The \( I_{P1} \) and \( I_{P2} \) denote the in-phase prompt accumulated correlation results over time epoch \( k-1 \) to \( k \) and \( k \) to \( k+1 \); \( Q_{P1} \) and \( Q_{P2} \) denote the quadrature prompt correlation results over time periods.

III. EXPERIMENTS

A. Experiment Setup

The experiment setup is shown in Fig. 3. The simulation is conducted using the trajectories generated from the IFEN hardware GPS RF signal simulator NavX®-NCS. The high frequency GPS signals are sampled, down converted and collected by the hardware front end GN3Sv3 from Sparkfun at the sampling frequency of 38.184 kHz. Then the intermediate frequency (IF) signals are handled over to the integration algorithm as the GPS signal input for post processing. The IMU data is generated using INS Toolbox 2.0 (2005) for Matlab developed by GPSSoft Ltd. The free Flight Dynamic Control Toolbox is used to calculate the initial attitudes of the platform based on the simulated dynamic trajectory.

Fig. 3. Simulation experiment setup.

The IMU data is simulated based on the parameters from a Landmark™ eXT MEMS-IMU. The main specification parameters are given in Table 1.
B. Straight Line Path

In the first experiment, a straight line trajectory is generated by the signal simulator with constant velocity of 110 m/s toward east. For comparison purpose, red curve of Fig. 4 is the navigation results from a deeply-coupled system. The blue dots are the standalone GPS solution using the least-squares estimator.

As shown in Fig. 4, the plot is given in ENU (east-north-up) frame. The deeply-coupled system presents much better navigation solution than the standalone GPS solution. This is because that the IMU estimates can smooth the noisy code phase and frequency errors, which stabilizes the tracking loop. The prompt in-phase components from 6 tracking loops are plotted in Fig. 5 to show the internal working of the tracking. The similar smoothed behaviour can also be observed in the velocity solution. Fig. 6 shows that the velocity of the object is also correctly estimated. The estimated east velocity is about 110 m/s, where in the north and up directions, the estimates are around zero.

Table I. Specification of LANDMARK<sup>TM</sup> 20 EXT MEMS-IMU

<table>
<thead>
<tr>
<th>Gyroscope (Angular rates)</th>
<th>Bias in-run Stability (1σ)</th>
<th>Noise(ARW) (1σ)</th>
<th>Scale factor error [ppm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20 [°/h]</td>
<td>0.035 [°/s/√Hz]</td>
<td>≤1000</td>
</tr>
<tr>
<td>Accelerometer (Specific forces)</td>
<td>Bias in-run Stability (1σ)</td>
<td>Noise(ARW) (1σ)</td>
<td>Scale factor error [ppm]</td>
</tr>
<tr>
<td></td>
<td>20 [μg]</td>
<td>40 [μg/√Hz]</td>
<td>≤1000</td>
</tr>
</tbody>
</table>

The results of straight line test show that the deeply-coupled integration system gives smoothed results in constant acceleration scenario. This is because a Kalman filter is used inside which fuses information from all the channels by estimating the common state, and the inertial sensors can provide dynamic measurements (in this test, they are zeros plus sensor errors), which are used in the system process model. Practically, using true dynamic measurements turns out to be better than using dynamic models. Because in reality, the true dynamic of the platform may often not be described by any model due to the lack of enough <i>a priori</i> knowledge on the trajectory.

For testing the algorithm with changing accelerations, a clockwise circle path is employed in the next section.

C. Clockwise Circle Path

In this test, a clockwise circle path is simulated. The platform flies with constant norm velocity of 30 m/s. The path radius is 1 km, with circle center point 48.1815° (latitude), 11.808° (longitude), 525 m (height). In this case, in the horizontal plane, the accelerations (i.e., the output of x and y-axis accelerometers) are changing over time. For the attitude, the heading (i.e., the output of gyro-z axis) is changing constantly over time. The position and velocity navigation results for 20 seconds are compared with GPS alone solution.

Fig. 5 shows that with the aiding from IMU observable, the tracking loops work fine. In this way, all the channels are connected to each other by the navigation filter, and each channel is able to correctly track the GPS satellite signals and decode the navigation data. The position and velocity navigation results for 20 seconds are compared with GPS alone solution.

The results of clockwise circle path test show that the deeply-coupled system gives better results in constant acceleration scenario. This is because the Kalman filter is used inside which fuses information from all the channels by estimating the common state, and the inertial sensors can provide dynamic measurements (in this test, they are zeros plus sensor errors), which are used in the system process model. Practically, using true dynamic measurements turns out to be better than using dynamic models. Because in reality, the true dynamic of the platform may often not be described by any model due to the lack of enough <i>a priori</i> knowledge on the trajectory.

For testing the algorithm with changing accelerations, a clockwise circle path is employed in the next section.
(using a least-squares estimator) in Fig. 7 and Fig. 8. In both figures, the red curves are the results from the deeply-coupled integration, while the blue curves are the results from GPS stand-alone system. The figures show that with INS aiding, the integration solution is less noisy and much smoother as compared with a GPS only solution.

leaves the outage environments. The proof of these benefits goes to the future work.

IV. CONCLUSION

In this paper, we have presented an implementation of an algorithm for a deeply-coupled GPS/INS system. The system models, using quaternions as the representation of attitude, are given. An extended Kalman filter is employed as the tool to fuse the data from the GPS receiver tracking loop outputs and IMU observables. In our system, the separated tracking channels are connected with each other due to the fact that global navigation filter estimates the common states. In this way, the channels ‘can help each other’ in the tracking which is a big difference from the tightly-coupled system. Besides, the IMU can capture the dynamics of the platform to improve the navigation solution for example in challenging GPS signal environment, and to enhance the system robustness. Using even a cheap IMU is better than using dynamic models in the Kalman filter, as in reality. The dynamics of the platform can often not be accurately described by any models. Two simulations have been made to verify the approach.

ACKNOWLEDGMENT

The first author gratefully acknowledges the support provided by the programme -Multi Modal Sensor Systems for Environmental Exploration and Safety (MOSES) within the Center for Sensor Systems (ZESS), University of Siegen, Germany.

REFERENCES